CS 473 ✧ Spring 2016

Homework 7

Due Tuesday, March 29, 2016, at 8pm

This is the last homework before Midterm 2.

---

1. Suppose we are given a two-dimensional array $A[1..m, 1..n]$ of non-negative real numbers. We would like to *round* A to an integer matrix, by replacing each entry x in A with either $\lfloor x \rfloor$ or $\lceil x \rceil$, without changing the sum of entries in any row or column of A. For example:

$$\begin{bmatrix} 1.2 & 3.4 & 2.4 \\ 3.9 & 4.0 & 2.1 \\ 7.9 & 1.6 & 0.5 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 4 & 2 \\ 4 & 4 & 2 \\ 8 & 1 & 1 \end{bmatrix}$$

   Describe and analyze an efficient algorithm that either rounds A in this fashion, or reports correctly that no such rounding exists.

2. You're organizing the Third Annual UIUC Computer Science 72-Hour Dance Exchange, to be held all day Friday, Saturday, and Sunday in Siebel Center.[1] Several 30-minute sets of music will be played during the event, and a large number of DJs have applied to perform. You need to hire DJs according to the following constraints.

   - Exactly k sets of music must be played each day, and thus $3k$ sets altogether.
   - Each set must be played by a single DJ in a consistent musical genre (ambient, bubblegum, dancehall, horrorcore, trip-hop, Nashville country, Chicago blues, axé, laïkó, skiffle, shape note, Nitzhonot, J-pop, K-pop, C-pop, T-pop, 8-bit, Tesla coil, ...).
   - Each genre must be played at most once per day.
   - Each DJ has given you a list of genres they are willing to play.
   - No DJ can play more than five sets during the entire event.

   Suppose there are n candidate DJs and g different musical genres available. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the $3k$ sets, or correctly reports that no such assignment is possible.

3. Describe and analyze an algorithm to determine, given an undirected[2] graph $G = (V, E)$ and three vertices $u, v, w \in V$ as input, whether G contains a simple path from u to w that passes through v.

---

[1] Efforts to secure overflow space in ECEB were sadly unsuccessful.

[2] This adjective is important; if the input graph were directed, this problem would be NP-hard.