New CS 473: Algorithms, Spring 2015
Homework 11

This homework will not be graded.

1. In Homework 10, we considered several different problems that can be solved by reducing them to a linear programming problem:

   - Finding a line that fits a given set of n points in the plane with minimum L_1 error.
   - Finding a line that fits a given set of n points in the plane with minimum L_∞ error.
   - Finding the largest matching in a bipartite graph.
   - Finding the smallest vertex cover in a bipartite graph.

   The specific linear programs are described in the homework solutions. For each of these linear programs, answer the following questions in the language of the original problem:

   (a) What is a basis?

   (b) (For the line-fitting problems only:) How many different bases are there?

   (c) What is a *feasible* basis?

   (d) What is a *locally optimal* basis?

   (e) What is a pivot?

2. Let $G = (V, E)$ be an arbitrary directed graph with weighted vertices; vertex weights may be positive, negative, or zero. A ***prefix*** of G is a subset $P \subseteq V$, such that there is no edge $u \to v$ where $u \notin P$ but $v \in P$. A ***suffix*** of G is the complement of a prefix. Finally, an ***interval*** of G is the intersection of a prefix of G and a suffix of G. The weight of a prefix, suffix, or interval is the sum of the weights of its vertices.

   (a) Describe a linear program that characterizes the maximum-weight prefix of G. Your linear program should have one variable per vertex, indicating whether that vertex is or is not in the chosen prefix.

   (b) Describe a linear program that characterizes the maximum-weight interval of G.

   [Hint: Don't worry about the solutions to your linear programs being integral; they will be. If all vertex weights are negative, the maximum-weight interval is empty; if all vertex weights are positive, the maximum-weight interval contains every vertex.]