CS 473: Undergraduate Algorithms, Spring 2010
Homework 2

Written solutions due Tuesday, February 9, 2010 at noon

- Roughly 1/3 of the students will give oral presentations of their solutions to the TAs. ***You should have received an email telling you whether you are expected to present this homework.*** Please see the course web page for further details.
- Groups of up to three students may submit a common solution. Please clearly write every group member's name and NetID on every page of your submission. Please start your solution to each numbered problem on a new sheet of paper. Please ***don't*** staple solutions for different problems together.

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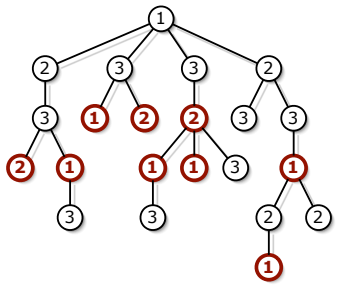
1. A ***palindrome*** is a string that reads the same forwards and backwards, like `x`, `pop`, `noon`, `redivider`, or `"sator arepo tenet opera rotas"`, Describe and analyze an algorithm to find the length of the longest subsequence of a given string that is also a palindrome. For example, the longest palindrome subsequence of **M**A**H**D**Y**NA**M**ICP**RO**G**R**AMZLET**M**ESHOW**Y**OUT**H**E**M** is MHYMRORMYHM, so given that string as input, your algorithm should output the number 11.

2. Oh, no! You have been appointed as the gift czar for Giggle, Inc.'s annual mandatory holiday party! The president of the company, who is certifiably insane, has declared that every Giggle employee must receive one of three gifts: (1) an all-expenses-paid six-week vacation anywhere in the world, (2) an all-the-pancakes-you-can-eat breakfast for two at Jumping Jack Flash's Flapjack Stack Shack, or (3) a burning paper bag full of dog poop. Corporate regulations prohibit any employee from receiving the same gift as his/her direct supervisor. Any employee who receives a better gift than his/her direct supervisor will almost certainly be fired in a fit of jealousy. How do you decide what gifts everyone gets if you want to minimize the number of people that get fired?

   More formally, suppose you are given a rooted tree T, representing the company hierarchy. You want to label each node in T with an integer 1, 2, or 3, such that every node has a different label from its parent.. The *cost* of an labeling is the number of nodes that have smaller labels than their parents. Describe and analyze an algorithm to compute the minimum cost of any labeling of the given tree T. (Your algorithm does *not* have to compute the actual best labeling—just its cost.)



A tree labeling with cost 9. Bold nodes have smaller labels than their parents.
This is ***not*** the optimal labeling for this tree.

3. After graduating from UIUC, you have decided to join the Wall Street Bank ***Boole Long Live***. The managing director of the bank, Eloob Egroeg, is a genius mathematician who worships George Boole[1] every morning before leaving for the office. The first day of every hired employee is a 'solve-or-die' day where s/he has to solve one of the problems posed by Eloob within 24 hours. Those who fail to solve the problem are fired immediately!

   Entering into the bank for the first time, you notice that the offices of the employees are organized in a straight row, with a large "T" or "F" written on the door of each office. Furthermore, between each adjacent pair of offices, there is a board marked by one of the symbols $\wedge, \vee$, or $\oplus$. When you ask about these arcane symbols, Eloob confirms that T and F represent the boolean values 'true' and 'false', and the symbols on the boards represent the standard boolean operators AND, OR, and XOR. He also explains that these letters and symbols describe whether certain combinations of employees can work together successfully. At the start of any new project, Eloob hierarchically clusters his employees by adding parentheses to the sequence of symbols, to obtain an unambiguous boolean expression. The project is successful if this parenthesized boolean expression evaluates to T.

   For example, if the bank has three employees, and the sequence of symbols on and between their doors is $T \wedge F \oplus T$, there is exactly one successful parenthesization scheme: $(T \wedge (F \oplus T))$. However, if the list of door symbols is $F \wedge T \oplus F$, there is no way to add parentheses to make the project successful.

   Eloob finally poses your solve-or-die question: Describe and algorithm to decide whether a given sequence of symbols can be parenthesized so that the resulting boolean expression evaluates to T. The input to your algorithm is an array $S[0..2n]$, where $S[i] \in \{T, F\}$ when i is even, and $S[i] \in \{\vee, \wedge, \oplus\}$ when i is odd.

[1] 1815-1864, The inventor of Boolean Logic