CS 473: Undergraduate Algorithms, Spring 2009
HBS 8

1. A box i can be specified by the values of its sides, say (i_1, i_2, i_3). We know all the side lengths are larger than 10 and smaller than 20 (i.e. $10 < i_1, i_2, i_3 < 20$). Geometrically, you know what it means for one box to nest in another: It is possible if you can rotate the smaller so that it fits inside the larger in each dimension. Of course, nesting is recursive, that is if i nests in j and j nests in k then i nests in k. After doing some nesting operations, we say a box is visible if it is not nested in any other one. Given a set of boxes (each specified by the lengthes of their sides) the goal is to find a set of nesting operations to minimize the number of visible boxes. Design and analyze an efficient algorithm to do this.

2. Let the number of papers submitted to a conference be n and the number of available reviewers be m. Each reviewer has a list of papers that he/she can review and each paper should be reviewed by three different persons. Also, each reviewer can review at most 5 papers. Design and analyze an algorithm to make the assignment or decide no feasible assignment exists.

3. Back in the euphoric early days of the Web, people liked to claim that much of the enormous potential in a company like Yahoo! was in the "eyeballs" - the simple fact that it gets millions of people looking at its pages every day. And further, by convincing people to register personal data with the site, it can show each user an extremely targeted advertisement whenever he or she visits the site, in away that TV networks or magazines could not hope to match. So if the user has told Yahoo! that he is a 20-year old computer science major from Cornell University, the site can throw up a banner ad for apartments in Ithaca, NY; on the other hand, if he is a 50-year-old investment banker from Greenwich, Connecticut, the site can display a banner ad pitching Lincoln Town Cars instead.

   But deciding on which ads to show to which people involves some serious computation behind the scenes. Suppose that the managers of a popular Web site have identified k distinct demographic groups $G_1, G_2, \ldots, G_k$. (These groups can overlap; for example G_1 can be equal to all residents of New York State, and G_2 can be equal to all people with a degree in computer science.) The site has contracts with m different advertisers, to show a certain number of copies of their ads to users of the site. Here is what the contract with the ith advertiser looks like:

   (a) For a subset $X_i \subset \{G_1, \ldots, G_k\}$ of the demographic groups, advertiser i wants its ads shown only to users who belong to at least one of the demographic groups in the set X_i

   (b) For a number r_i, advertiser i wants its ads shown to at least r_i users each minute.

   Now, consider the problem of designing a good advertising policy - a way to show a single ad to each user of the site. Suppose at a given minute, there are n users visiting the site. Because we have registration information on each of these users, we know that user j (for $j = 1, 2, \ldots, n$) belongs to a subset $U_j \subset \{G_1, \ldots, G_k\}$ of the demographic groups. The problem is: is there a way to show a single ad to each user so that the site's contracts with each of the m advertisers is satisfied for this minute? (That is, for each $i = 1, 2, \ldots, m$, at least r_i of the n users, each belonging to at least one demographic group in X_i, are shown an ad provided by advertiser i.)

   Give an efficient algorithm to decide if this is possible, and if so, to actually choose an ad to show each user.