## CS 473: Undergraduate Algorithms, Spring 2009 HBS 4

1. Let $x$ and $y$ be two elements of a set $S$ whose ranks differ by exactly $r$. Prove that in a treap for $S$, the expected length of the unique path from $x$ to $y$ is $O(\log r)$
2. Consider the problem of making change for $n$ cents using the least number of coins.
(a) Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.
(b) Suppose that the available coins have the values $c^{0}, c^{1}, \ldots, c^{k}$ for some integers $c>1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.
(c) Give a set of 4 coin values for which the greedy algorithm does not yield an optimal solution, show why.
(d) Give a dynamic programming algorithm that yields an optimal solution for an arbitrary set of coin values.
3. A heater is a sort of dual treap, in which the priorities of the nodes are given, but their search keys are generate independently and uniformly from the unit interval [0,1]. You can assume all priorities and keys are distinct. Describe algorithms to perform the operations INSERT and DELETEMIN in a heater. What are the expected worst-case running times of your algorithms? In particular, can you express the expected running time of INSERT in terms of the priority rank of the newly inserted item?
