CS 473: Undergraduate Algorithms, Spring 2009 HBS 4

- 1. Let *x* and *y* be two elements of a set *S* whose ranks differ by exactly *r*. Prove that in a treap for *S*, the expected length of the unique path from *x* to *y* is $O(\log r)$
- 2. Consider the problem of making change for *n* cents using the least number of coins.
 - (a) Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.
 - (b) Suppose that the available coins have the values c⁰, c¹,...,c^k for some integers c > 1 and k ≥ 1. Show that the greedy algorithm always yields an optimal solution.
 - (c) Give a set of 4 coin values for which the greedy algorithm does not yield an optimal solution, show why.
 - (d) Give a dynamic programming algorithm that yields an optimal solution for an arbitrary set of coin values.
- 3. A heater is a sort of dual treap, in which the priorities of the nodes are given, but their search keys are generate independently and uniformly from the unit interval [0,1]. You can assume all priorities and keys are distinct. Describe algorithms to perform the operations INSERT and DELETEMIN in a heater. What are the expected worst-case running times of your algorithms? In particular, can you express the expected running time of INSERT in terms of the priority rank of the newly inserted item?