

For each of the following languages over the alphabet  $\Sigma = \{0, 1\}$ , either prove the language is regular (by giving an equivalent regular expression, DFA, or NFA) or prove that the language is not regular (using a fooling set argument). Exactly half of these languages are regular.

1.  $\{0^n 1 0^n \mid n \geq 0\}$
2.  $\{0^n 1 0^n w \mid n \geq 0 \text{ and } w \in \Sigma^*\}$
3.  $\{w 0^n 1 0^n x \mid w \in \Sigma^* \text{ and } n \geq 0 \text{ and } x \in \Sigma^*\}$
4. Strings in which the number of 0s and the number of 1s differ by at most 2.
5. Strings such that *in every prefix*, the number of 0s and the number of 1s differ by at most 2.
6. Strings such that *in every substring*, the number of 0s and the number of 1s differ by at most 2.