

Describe deterministic finite-state automata that accept each of the following languages over the alphabet  $\Sigma = \{0, 1\}$ . You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states  $Q$ , the start state  $s$ , the accept states  $A$ , and the transition function  $\delta$  are all be clear. Try to keep the number of states small.

1. All strings in which the number of **0**s is even and the number of **1**s is *not* divisible by 3.
2. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

For example, the string **1100** is an element of this language, because it represents  $2^3 + 2^2 = 12$  in binary and  $3^3 + 3^2 = 36$  in ternary.

**Work on these later:**

3. All strings  $w$  such that  $\binom{|w|}{2} \bmod 6 = 4$ . [Hint: Maintain both  $\binom{|w|}{2} \bmod 6$  and  $|w| \bmod 6$ .]
- \*4. All strings  $w$  such that  $F_{\#(\mathbf{10}, w)} \bmod 10 = 4$ , where  $\#(\mathbf{10}, w)$  denotes the number of times **10** appears as a substring of  $w$ , and  $F_n$  is the  $n$ th Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$