"CS 374" Fall 2014 — Homework 9
Due Tuesday, November 18, 2014 at noon

The following questions ask you to describe various Turing machines. In each problem, give *both* a formal description of your Turing machine in terms of specific states, tape symbols, and transition functions *and* explain in English how your Turing machine works. In particular:

- Clearly specify what variant of Turing machine you are using: Number of tapes, number of heads, allowed head motions, halting conditions, and so on.

- Include the type signature of your machine's transition function. The standard model uses a transition function whose signature is $\delta : Q \times \Gamma \to Q \times \Gamma \times \{-1, +1\}$.

- If necessary, break your Turing machine into smaller functional pieces, and describe those pieces separately (both formally and in English).

- Use state names that convey their meaning/purpose.

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1. Describe a Turing machine that computes the function $\lceil \log_2 n \rceil$. Given the string 1^n as input, for any positive integer n, your machine should return the string $1^{\lceil \log_2 n \rceil}$ as output. For example, given the input string 1111111111111 (thirteen 1s), your machine should output the string 1111, because $2^3 < 13 \le 2^4$.

2. A **binary-tree** Turing machine uses an infinite binary tree as its tape; that is, *every* cell in the tape has a left child and a right child. At each step, the head moves from its current cell to its *P*arent, its *L*eft child, or to its *R*ight child. Thus, the transition function of such a machine has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{P, L, R\}$. The input string is initially given along the left spine of the tape.

   Prove that any binary-tree Turing machine can be simulated by a standard Turing machine. That is, given any binary-tree Turing machine $M = (\Gamma, \square, \Sigma, Q, \text{start}, \text{accept}, \text{reject}, \delta)$, describe a standard Turing machine $M' = (\Gamma', \square', \Sigma, Q', \text{start}', \text{accept}', \text{reject}', \delta')$ that accepts and rejects exactly the same input strings as M. Be sure to describe how a single transition of M is simulated by M'.

   In addition to submitting paper solutions, please also electronically submit your solution to this problem on CrowdGrader.

3. **[Extra credit]**

   A **tag**-Turing machine has two heads: one can only read, the other can only write. Initially, the read head is located at the left end of the tape, and the write head is located at the first blank after the input string. At each transition, the read head can either move one cell to the right or stay put, but the write head *must* write a symbol to its current cell and move one cell to the right. Neither head can ever move to the left.

   Prove that any standard Turing machine can be simulated by a tag-Turing machine. That is, given any standard Turing machine M, formally describe a tag-Turing machine M' that accepts and rejects exactly the same strings as M. Be sure to describe how a single transition of M is simulated by M'.

CS 374 Fall 2014 — Homework 9 Problem 1

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Formally describe a Turing machine that computes the function $\lceil \log_2 n \rceil$.

CS 374 Fall 2014 — Homework 9 Problem 2

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Prove that any binary-tree Turing machine can be simulated by a standard Turing machine.

CS 374 Fall 2014 — Homework 9 Problem 3

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[Extra credit] Prove that any standard Turing machine can be simulated by a tag-Turing machine.