1. A set of vectors A is said to be linearly independent if no $v \in A$ can be expressed as a linear combination of the vectors in $A - \{v\}$. Given a set of vectors S, describe an efficient algorithm for finding a linearly independent subset of S with the maximum possible size. Assume you are given a function that can check if n vectors are linearly independent in $O(n^2)$ time.

2. You live in a country with n different types of coins, with values $1, 2, 2^2, ..., 2^{n-1}$. Describe an efficient algorithm for determining how to make change for a given value W using the least possible number of coins.

3. Let X be a set of n intervals on the real line. A *proper coloring* of X assigns a color to each interval, so that any two overlapping intervals are assigned different colors. Describe an efficient algorithm to compute the minimum number of colors needed to properly color X. Assume that your input consists of two array $L[1..n]$ and $R[1..n]$, where $L[i]$ and $R[i]$ are the left and right endpoints of the ith interval.