

**1. NP-hardness Proofs: Restriction**

Prove that each of the following problems is NP-hard. In each part, find a special case of the given problem that is equivalent to a known NP-hard problem.

**(a) Longest Path**

Given a graph  $G$  and a positive integer  $k$ , does  $G$  contain a path with  $k$  or more edges?

**(b) Partition into Hamiltonian Subgraphs**

Given a graph  $G$  and a positive integer  $k$ , can the vertices of  $G$  be partitioned into at most  $k$  disjoint sets such that the graph induced by each set has a Hamiltonian cycle?

**(c) Set Packing**

Given a collection of finite sets  $C$  and a positive integer  $k$ , does  $C$  contain  $k$  disjoint sets?

**(d) Largest Common Subgraph**

Given two graphs  $G_1$  and  $G_2$  and a positive integer  $k$ , does there exist a graph  $G_3$  such that  $G_3$  is a subgraph of both  $G_1$  and  $G_2$  and  $G_3$  has at least  $k$  edges?

**2. Domino Line**

You are given an unusual set of dominoes; each domino has a number on each end, but the numbers may be arbitrarily large and some numbers appear on many dominoes, while other numbers only appear on a few dominoes. Your goal is to form a line using all the dominoes so that adjacent dominoes have the same number on their adjacent halves. Either give an efficient algorithm to solve the problem or show that it is NP-hard.

**3. Set Splitting**

Given a finite set  $S$  and a collection of subsets  $C$  is there a partition of  $S$  into two sets  $S_1$  and  $S_2$  such that no subset in  $C$  is contained entirely in  $S_1$  or  $S_2$ ? Show that the problem is NP-hard. (Hint: use NAE-3SAT, which is similar to 3SAT except that a satisfying assignment does not allow all 3 variables in a clause to be true.)