1. **Self-reductions**

   In each case below assume that you are given a black box which can answer the decision version of the indicated problem. Use a polynomial number of calls to the black box to construct the desired set.

   (a) Independent set: Given a graph G and an integer k, does G have a subset of k vertices that are pairwise nonadjacent?

   (b) Subset sum: Given a multiset (elements can appear more than once) $X = \{x_1, x_2, \ldots, x_k\}$ of positive integers, and a positive integer S does there exist a subset of X with sum exactly S?

2. **Lower Bounds**

   Give adversary arguments to prove the indicated lower bounds for the following problems:

   (a) Searching in a sorted array takes at least $1 + \lfloor \lg_2 n \rfloor$ queries.

   (b) Let M be an $n \times n$ array of real values that is increasing in both rows and columns. Prove that searching for a value requires at least n queries.

3. **k-coloring**

   Show that we can solve the problem of constructing a k-coloring of a graph by using a polynomial number of calls to a black box that determines whether a graph has such a k-coloring. (Hint: Try reducing via an intermediate problem that asks whether a partial coloring of a graph can be extended to a proper k-coloring.)