1. The Acme Company is planning a company party. In planning the party, each employee is assigned a fun value (a positive real number). The goal of the party planners is to maximize the total fun value (sum of the individual fun values) of the employees invited to the party. However, the planners are not allowed to invite both an employee and his direct boss. Given a tree containing the boss/underling structure of Acme, find the invitation list with the highest allowable fun value.
2. An inversion in an array $A$ is a pair $i, j$ such that $i<j$ and $A[i]>A[j]$. (In an $n$-element array, the number of inversions is between 0 and $\binom{n}{2}$.)
Find an efficient algorithm to count the number of inversions in an $n$-element array.
3. A tromino is a geometric shape made from three squares joined along complete edges. There are only two possible trominoes: the three component squares may be joined in a line or an L-shape.
(a) Show that it is possible to cover all but one square of a $64 \times 64$ checkerboard using L-shape trominoes. (In your covering, each tromino should cover three squares and no square should be covered more than once.)
(b) Show that you can leave any single square uncovered.
(c) Can you cover all but one square of a $64 \times 64$ checkerboard using line trominoes? If so, which squares can you leave uncovered?
